# On the higher-derivative supersymmetric gauge theory

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We study the one-loop low-energy effective action for the higher-derivative superfield gauge theory coupled to a chiral matter.

## I.   INTRODUCTION

The use of higher-derivatives has been proposed as a way to tame the ultraviolet behavior of physically relevant models. Actually, a finite version of QED was put forward by Lee and Wick about fourty years ago [1]; that proposal was nevertheless beset by the presence of spurious degrees of freedom which induce indefinite metric in the space of states jeopardizing unitarity and so required special treatment. After a long story, recently the idea was revived in the so-called Lee-Wick standard model leading to some new insights in the hierachy problem [2]. Furthermore, following a similar thread, higher-curvature gravitational models were considered; in spite of the possible breaking of unitarity, they furnish renormalizable quantum gravity models [3] which may be useful for cosmological applications (for recent review work in this direction see [4]).

In supersymmetric models, a higher-derivative regularization method was proposed in [5]. Further, interest in the higher-derivative supersymmetric field theories increased not only due to their application within the regularization context (see [6] for many examples) but also in other contexts. For example, the higher-derivative supergravity model, which, in the

superconformal sector, can be treated as a natural higher-derivative generalization of the Wess-Zumino model, has been studied in [7, 8]. Afterwards, some classical aspects of the very generic class of the higher-derivative chiral superfield models have been considered in [9], and the lower perturbative corrections in such theories were obtained in [10].

Therefore, the natural continuation of these studies could be the construction of a consistent higher-derivative gauge theory coupled to a chiral matter. Within this paper, we construct such a model (unfortunately, this construction is probably possible only in the Abelian case). We calculate the one-loop low-energy effective action for such a theory generalizing the results of [11] where the usual super-Yang-Mills field coupled to a chiral matter has been studied in the one-loop approximation. Within this study, we employ the superfield approach for calculating the supersymmetric effective potential developed in Refs. [12–14].

## II.   EFFECTIVE ACTION IN HIGHER-DERIVATIVE SUPERFIELD GAUGE THEORIES: GENERAL APPROACH

Let us formulate the supersymmetric higher-derivative gauge theory. It is well known [15] that under the usual gauge transformation of the gauge superfield $v$

$$e^{gv} \to e^{-ig\bar{\Lambda}}e^{gv}e^{ig\Lambda}, \tag{1}$$

where $\Lambda$ is a chiral parameter, and $\bar{\Lambda}$ is an antichiral one, the superfield strength

$$W_\alpha = \bar{D}^2(e^{-gv}D_\alpha e^{gv}), \tag{2}$$

is transformed as

$$W_\alpha \to e^{-ig\Lambda}W_\alpha e^{ig\Lambda}. \tag{3}$$

In the Abelian case, the strength $W_\alpha$ is invariant.

Now, let us try to implement the higher derivatives in the super-Yang-Mills theory with matter whose standard form is

$$S = \int d^8z \bar{\phi}e^{gV}\phi + \frac{1}{2g^2}\mathrm{tr}\int d^6z W^\alpha W_\alpha. \tag{4}$$

In the non-Abelian case, the $\phi$ is a column vector (we can consider as well the case when the $\phi$ is not an isospinor but also the Lie-algebra valued field, as $W^\alpha$ is; however, it is not

relevant within our approach). The first idea consists in inserting of some operator in the purely gauge part of the action rewriting it as

$$S_W = \frac{1}{2g^2} \text{tr} \int d^6 z W^\alpha \mathcal{O} W_\alpha. \tag{5}$$

In the non-Abelian case, this action is invariant under these gauge transformations if and only if the operator $\mathcal{O}$ satisfies the condition

$$e^{ig\Lambda} \mathcal{O} e^{-ig\Lambda} = \mathcal{O}. \tag{6}$$

Expanding this equation order by order in $g$, we find that this condition can be satisfied only if $[\Lambda, \mathcal{O}] = 0$, i.e. the operator $\mathcal{O}$ is proportional to the unit operator, and neither spinor, nor vector derivatives are allowed. Therefore, in the non-Abelian case, higher derivatives cannot be introduced in a manner compatible with the gauge invariance.

So, we will restrict ourselves to develop the Abelian case where the gauge transformations are reduced to [15]

$$v \to v + i(\Lambda - \bar{\Lambda}), \tag{7}$$

and the strength $W_\alpha$ is explicitly gauge invariant. In this case, as a first, simplest way, we can write the following higher-derivative action for the Abelian gauge superfield:

$$S_W = \frac{1}{2g^2} \int d^6 z W^\alpha (\Box + m^2) W_\alpha = -\frac{1}{16} \int d^8 z v D^\alpha \bar{D}^2 D_\alpha (\Box + m^2) v. \tag{8}$$

We note that this action (but with $m = 0$) was earlier [8] employed as an auxiliary tool for studying the higher-derivative generalization of the Wess-Zumino model. However, the detailed studies of the properties of this action were not carried out. Now, we should develop now its consistent coupling to the chiral matter. It is clear that this coupling is invariant if we suppose that the chiral and antichiral superfields are transformed as

$$\phi \to e^{-i\Lambda} \phi, \quad \bar{\phi} \to \bar{\phi} e^{i\bar{\Lambda}}. \tag{9}$$

A natural generalization of the chiral superfield action

$$S_\Phi = \int d^8 z \bar{\phi} e^{gV} \mathcal{R} \phi, \tag{10}$$

with $\mathcal{R}$ some operator, is gauge invariant again, if $\mathcal{R}$ is proportional to the unit operator, both in the Abelian and in the non-Abelian case. Therefore, the simplest action for the

higher-derivative supersymmetric gauge theory, that is, for the supersymmetric scalar QED is

$$S = \int d^8z \bar{\phi} e^{gV} \phi - \frac{1}{16} \int d^8z v D^\alpha \bar{D}^2 D_\alpha (\Box + m^2) v. \qquad (11)$$

In principle, this action can be generalized by introducing the some self-couplings of set of the chiral superfields (see f.e. [16]), however, we here want to study the generic structure for the one-loop low-energy effective action for the chiral superfield.

To fix the gauge, we add the gauge-fixing term:

$$S_{gf} = \frac{1}{16\alpha} \int d^8z v D^2 \bar{D}^2 (\Box + m^2) v. \qquad (12)$$

Here $\alpha$ is the gauge-fixing parameter.

Following [15], the low-energy effective action in the theory of chiral scalar superfield is described by the Kählerian effective potential which depends only on chiral and antichiral superfields but not on their derivatives. Namely this effective potential will be the principal object of study in the paper. We note that since the model (11) does not involve chiral self-coupling of the matter fields, the chiral effective potential in it will be identically equal to zero unlike in the Wess-Zumino model and other models where such a coupling is present. The ghosts are completely factorized since the theory is Abelian.

The propagators in the theory (11) are very similar to the propagators in the usual super-Yang-Mills theory [15]:

$$< \phi(z_1)\bar{\phi}(z_2) > = \frac{\bar{D}^2 D^2}{16\Box} \delta^8(z_1 - z_2); \quad < \bar{\phi}(z_1)\phi(z_2) > = \frac{D^2 \bar{D}^2}{16\Box} \delta^8(z_1 - z_2);$$

$$< v(z_1)v(z_2) > = -\frac{1}{\Box(\Box + m^2)} \left( -\frac{D^\alpha \bar{D}^2 D_\alpha}{8\Box} + \alpha \frac{\{\bar{D}^2, D^2\}}{16\Box} \right) \delta^8(z_1 - z_2). \qquad (13)$$

It is convenient to express these propagators in terms of the projecting operators [15]

$$\Pi_0 = \frac{\{\bar{D}^2, D^2\}}{16\Box}, \quad \Pi_{1/2} = -\frac{D^\alpha \bar{D}^2 D_\alpha}{8\Box}.$$

Indeed, it is clear that $\Pi_0^n = \Pi_0$, $\Pi_{1/2}^n = \Pi_{1/2}$ (for any integer $n \geq 1$), $\Pi_0 \Pi_{1/2} = \Pi_{1/2} \Pi_0 = 0$. Thus, we can write

$$< \phi(z_1)\bar{\phi}(z_2) > + < \bar{\Phi}(z_1)\Phi(z_2) > = \Pi_0 \delta^8(z_1 - z_2);$$

$$< v(z_1)v(z_2) > = -\frac{1}{\Box(\Box + m^2)} (\Pi_{1/2} + \alpha \Pi_0) \delta^8(z_1 - z_2). \qquad (14)$$

Here we emphasized the combination $< \Phi(z_1)\bar{\Phi}(z_2) > + < \bar{\Phi}(z_1)\Phi(z_2) >$ since namely it will arise in many cases including the contributions to the one-loop kählerian effective potential.

## III.   ONE-LOOP CALCULATIONS

Now, let us start with study of the one-loop kählerian potential. At the one-loop order, we will have two types of contributions. In the first of them, all diagrams involve only the gauge field propagators:
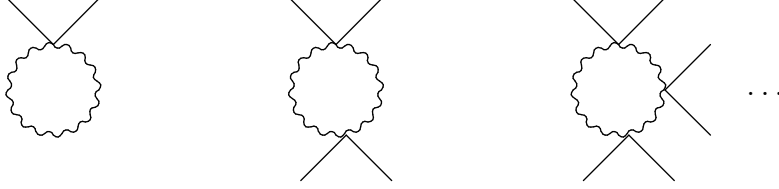


The contribution of the sum of these diagrams can be expressed as

$$K_a^{(1)} = \int d^8 z_1 \sum_{n=1}^{\infty} \frac{(-1)^n}{2n} (g^2 \Phi \bar{\Phi} \frac{1}{\Box(\Box + m^2)} (\Pi_{1/2} + \alpha \Pi_0))^n \delta_{12}|_{\theta_1 = \theta_2}, \tag{15}$$

where $\frac{1}{n}$ is a symmetry factor. The $\Phi$, $\bar{\Phi}$ are the background fields. These diagrams do not involve the triple vertices, only the quartic ones.

Using the properties of the projecting operators, we can write

$$K_a^{(1)} = \int d^8 z_1 \sum_{n=1}^{\infty} \frac{(-1)^n}{2n} (g^2 \Phi \bar{\Phi} \frac{1}{\Box(\Box + m^2)})^n (\Pi_{1/2} + \alpha^n \Pi_0)) \delta_{12}|_{\theta_1 = \theta_2}. \tag{16}$$

Since $\frac{D^2 \bar{D}^2}{16} \delta_{12} = 1$, we have $\Box \Pi_0 \delta_{12}|_{\theta_1 = \theta_2} = 2$, and $\Box \Pi_{1/2} \delta_{12}|_{\theta_1 = \theta_2} = -2$. Thus, we have

$$K_a^{(1)} = \int d^8 z_1 \sum_{n=1}^{\infty} \frac{(-1)^n}{n} \frac{1}{\Box} (g^2 \Phi \bar{\Phi} \frac{1}{\Box(\Box + m^2)})^n (1 - \alpha^n) \delta^4(x_1 - x_2)|_{x_1 = x_2}. \tag{17}$$

By carrying out the Fourier transform $\Box \to -k^2$ and Wick rotation $k_0 = ik_{0E}$ which yields $k^2 = k_E^2$, we arrive at

$$K_a^{(1)} = -i \int d^8 z \int \frac{d^4 k_E}{(2\pi)^4} \sum_{n=1}^{\infty} \frac{(-1)^n}{n} \frac{1}{\Box} (\frac{g^2 \Phi \bar{\Phi}}{k^2(k^2 + m^2)})^n (1 - \alpha^n). \tag{18}$$

Then, by using the expansion

$$\sum_{n=1}^{\infty} \frac{(-a)^n}{n} = -\ln(1 + a), \tag{19}$$

we have

$$K_a^{(1)} = i \int d^8 z \int \frac{d^4 k_E}{(2\pi)^4} \frac{1}{k^2} \left[ \ln(1 + \frac{g^2 \Phi \bar{\Phi}}{k^2(k^2 + m^2)}) - \ln(1 + \frac{\alpha g^2 \Phi \bar{\Phi}}{k^2(k^2 + m^2)}) \right]. \tag{20}$$

Notice that at $\alpha = 0$ (Landau gauge), the second term in this expression vanishes.

The second type of diagrams involves the triple vertices as well. We should first introduce a "dressed" propagator



In this propagator, the summation over all quartic vertices is performed. As a result, this "dressed" propagator is equal to

$$
\begin{aligned}
< vv >_D &= < vv > (1 + g^2 \Phi \bar{\Phi} < vv > + (g^2 \Phi \bar{\Phi} < vv >)^2 + \ldots) = \\
&= - \sum_{n=0}^{\infty} (g^2 \Phi \bar{\Phi})^n \frac{1}{(\Box(\Box + m^2))^{n+1}} (\Pi_{1/2} + \alpha \Pi_0)^{n+1}.
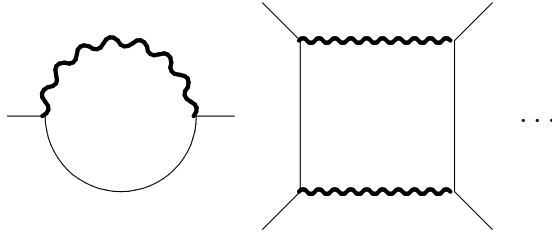\end{aligned}
\tag{21}
$$

Summing up, we arrive at

$$
< vv >_D = -\left( \frac{1}{\Box(\Box + m^2) + g^2 \Phi \bar{\Phi}} \Pi_{1/2} + \frac{\alpha}{\Box(\Box + m^2) + \alpha g^2 \Phi \bar{\Phi}} \Pi_0 \right) \delta^8(z_1 - z_2).
\tag{22}
$$

As a result, we should sum over diagrams representing themselves as cycles of all possible number of links each of which has the form
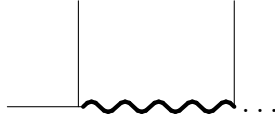


Such diagrams look like



The complete contribution of all these cycles looks like

$$
K_b^{(1)} = \int d^8 z_1 \sum_{n=1}^{\infty} \frac{1}{2n} (g^2 \Phi \bar{\Phi} (< \phi \bar{\phi} > + < \bar{\phi} \phi >) < vv >_D)^n \delta_{12}|_{\theta_1 = \theta_2},
\tag{23}
$$

or, as is the same,

$$
K_b^{(1)} = \int d^8 z_1 \sum_{n=1}^{\infty} \frac{1}{2n} (g^2 \Phi \bar{\Phi} \Pi_0 < vv >_D)^n \delta_{12}|_{\theta_1 = \theta_2}.
\tag{24}
$$

By noting that

$$
\Pi_0 < vv >_D = -\frac{\alpha}{\Box(\Box + m^2) + \alpha g^2 \Phi \bar{\Phi}} \Pi_0,
\tag{25}
$$

we can rewrite the expression above as

$$
K_b^{(1)} = \int d^8 z_1 \sum_{n=1}^{\infty} \frac{(-1)^n}{2n} \left( \frac{\alpha g^2 \Phi \bar{\Phi}}{\Box(\Box + m^2) + g^2 \Phi \bar{\Phi}} \right)^n \Pi_0 \delta_{12}|_{\theta_1 = \theta_2}.
\tag{26}
$$

Since $\square \Pi_0 \delta_{12}|_{\theta_1 = \theta_2} = 2$, we have

$$K_b^{(1)} = \int d^8 z_1 \sum_{n=1}^{\infty} \frac{1}{n} \frac{1}{\square} (-\frac{\alpha g^2 \Phi \bar{\Phi}}{\square(\square + m^2) + g^2 \Phi \bar{\Phi}})^n \delta^4(x_1 - x_2)|_{x_1 = x_2}. \tag{27}$$

Carrying out the Fourier transform, Wick rotation and summation as above, we arrive at

$$K_b^{(1)} = i \int d^8 z \int \frac{d^4 k_E}{(2\pi)^4} \frac{1}{k_E^2} \left[ \ln(1 + \frac{\alpha g^2 \Phi \bar{\Phi}}{k_E^2(k_E^2 + m^2)}) \right]. \tag{28}$$

Summing this contribution with $K_a^{(1)}$ (20), we see that the $\alpha$ dependent contribution vanishes, and the total one-loop kählerian effective potential is gauge independent, being, after returning to the Minkowski space, equal to

$$K^{(1)} = \int d^8 z \int \frac{d^4 k_E}{(2\pi)^4} \frac{1}{k_E^2} \ln \left[ 1 + \frac{g^2 \Phi \bar{\Phi}}{k_E^2(k_E^2 + m^2)} \right]. \tag{29}$$

This integral, after change $k^2 = u$ and $d^4 k = \pi^2 u du$, can be rewritten in spherical coordinates

$$K^{(1)} = \int d^8 z \int_0^{\infty} \frac{du}{(4\pi)^2} \ln \left[ 1 + \frac{g^2 \Phi \bar{\Phi}}{u(u + m^2)} \right]. \tag{30}$$

Let us calculate this integral. Although it can be found straightforwardly, it explicit form seems to be rather cumbersome. Thertefore it is instructive to estimate its value in some approximate situations. First, we suppose that the mass is small but non-zero. In this case, this integral can be approximately represented as

$$K^{(1)} = \int d^8 z \int_{m^2}^{\infty} \frac{du}{(4\pi)^2} \ln \left[ 1 + \frac{g^2 \Phi \bar{\Phi}}{u^2} \right], \tag{31}$$

which yields

$$K^{(1)} = \frac{1}{(4\pi)^2} \int d^8 z \left\{ (g^2 \Phi \bar{\Phi})^{1/2} \left[ \pi - 2 \arctan \left( \frac{m^2}{(g^2 \Phi \bar{\Phi})^{1/2}} \right) \right] - \frac{m^2}{2} \ln \left( 1 + \frac{g^2 \Phi \bar{\Phi}}{m^4} \right) \right\}. \tag{32}$$

Second, we suppose that the mass is zero. In this case we find

$$K^{(1)} = \frac{1}{16\pi} \int d^8 z (g^2 \Phi \bar{\Phi})^{1/2}. \tag{33}$$

It is worth to notice that both results (32) and (33) are finite and do not need any renormalization. In other words, the one-loop kählerian effective potential to the higher-derivative supersymmetric abelian gauge theory does not display any divergences, unlike the usual gauge theories [11].

## IV.  SUMMARY

We have explicitly found the one-loop kählerian effective potential in the supersymmetric higher-derivative QED. This term evidently dominates in the low-energy limit. We note that the chiral contributions to the effective action, which are known to be typical for the Wess-Zumino model and its straightforward many-field generalizations [12], simply do not arise due to the absence of the chiral self-interaction in the classical action. However, had we introduced such a term, it is most probably that the chiral contributions to the effective action could either display the infrared singularities due to the augmented degree of momenta in the denominator (compare with [17] for the usual supersymmetric QED), in the case when the higher-derivative theory is massless, or give zero result in the opposite case (one should remind that, in the Wess-Zumino model, the chiral effective potential also does not emerge in the massive case). The natural continuation of this study could consist in the formulation and study of more generic higher-derivative supersymmetric gauge models. We will return to this problem in a forthcoming paper.

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